

A Few Apparent or Just Seeming Problems with the K-Axiom of SDL in Attempted Applications to “Real-Life” Deontic Discourse

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Abstract: A Few Apparent or Just Seeming Problems with the K-Axiom of SDL in Attempted Applications to “Real-Life” Deontic Discourse

The objective of this paper is to define a few, amongst many, difficulties with making deontic logic in the established, classical form known as “Standard Deontic Logic” developed starting from Georg Henrik von Wright’s seminal paper of 1951, *Deontic Logic*, published in the *Mind* in 1951. Since then, there has been impressive development but even the “classical” logical systems (with later modifications) have gained little and next to none recognition as *instruments of controlling deontic discourse*. I try to find out the “culprits” of this state of affairs and attempt, on a small scale, to indicate remedies.

Keywords: Standard deontic logic, paradoxes of, applications in real-life deontic discourse; Kripke semantics; K-principle.

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1. Introduction

Logic matters, as P. T. Geach once put it. For all kinds of reasons. Yet, deontic logic¹ has not yet won due recognition from practising “deonticians” of any sort:

¹ By “deontic logic” I shall here be referring to what could be called “a series of footnotes to von Wright”, viz., to his G.H. von Wright, “Deontic Logic”, in *Mind*, 1951, 60(237), pp. 1–15; G.H. von Wright “A note on deontic logic and derived obligation”, in *Mind*, 1956, 65(1), pp. 507–509, leaving all the other systems, e.g., Mally’s, Becker’s, Kalinowski’s, or Weinberger’s aside. This logic is usually called “Standard Deontic Logic” (SDL) and is codified in P. McNamara, F. Van De Putte, “Deontic Logic”, in *The Stanford Encyclopedia of Philosophy* (Fall 2022 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2022/entries/logic-deontic/> [accessed 20/12/2022]. Generally, the present essay is motivated by the “spirit”, if not the “letter”

legal scholars, ethicists, specialists in etiquette (“good manners”), prudential considerations and the like. Books with titles like “Logic for lawyers” (such as Aldisert 1997²) hardly ever go beyond syllogistics and/or first order classical predicate calculus, and if deontic logic is mentioned in it at all, the mention is furnished with various kinds of rather off-putting warnings of the sort “deontic logic is difficult and abundant in paradoxes”³. As for philosophers who are not logicians, they often, like Charles Pigden, for instance, “take [...] a dim view of deontic logic” because “[m]any of its leading principles are false, bordering on the nonsensical, and even the reasonably plausible ones are subject to devastating counter-examples”⁴. Some legal philosophers seem to hold that deontic logic is unsuitable for checking the correctness of real life deontic reasonings, and set out to “*dénoncer les conceptions erronées auxquelles ces fonctions normatives ou modalités déontiques [deontic operators] ont donné lieu*”⁵. A candid confession such as

we should remember that [...] some formulas of deontic logic when they are taken out of their context and read in natural language appear paradoxical, whereas understood in the spirit of a logical system are non-controversial⁶.

does not help much (but it is good to know that some deontic logicians are at all aware of the problem).

In a less sweepingly critical, not to say disparaging key, the postulate is sometimes raised that deontic logic should be not just formally correct⁷, but also “materially adequate”⁸, that is, should be up to the task of “collecting our

of O. Weinberger, “Normenlogik anwendbar im Recht”, in *Logique et Analyse*, 1970, 13(49/50), pp. 93–106, although the size, meaning and calibre of Weinberger’s work cannot but put the present author to shame.

² R.J. Aldisert, *Logic for Lawyers: A Guide to Clear Legal Thinking*, National Institute for Trial Advocacy, South Bend (Ind.), 1997.

³ One of the few exceptions is Z. Ziemiński, *Practical Logic*, Springer, Dordrecht, 1976, with the appendix on deontic logic by Ziemia. See Z. Ziemia, “Deontic Logic”, in Z. Ziemiński, *Practical Logic, cit.*, pp. 360–430. Yet my impression is that the latter might be too advanced and sophisticated for the intended reader.

⁴ C.R. Pigden, “Introduction”, in C. R. Pigden (ed.), *Hume on Motivation and Virtue*, Palgrave Macmillan, London, 2009, p. 30.

⁵ P. Amselek, “Les fonctions normatives ou catégories modales”, in *Philosophiques*, 2006, 33(2), pp. 391–418, p. 391.

⁶ R. Trypuz, P. Kulicki, “Jerzy Kalinowski’s Logic of Normative Sentences Revisited”, in *Studia Logica*, 2015, 103(2), p. 392; P. McNamara, F. Van De Putte, “Deontic Logic”, *cit., ad init.*, say that “we need to be cautious about making too easy a link between deontic logic and practicality”. We are, I think, and for this reason SLD “is the most [...] studied system of deontic logic” but seldom if ever applied one, though probably more often than others.

⁷ I.e., among other things, have univocal rules of expression formation, comprise an explanation of what its variables range over (sentences? propositions? states of affairs? events? actions? types of actions?), be consistent and the like.

⁸ G. Kalinowski, “Norms and Logic”, in *American Journal of Jurisprudence*, 1973, 18(1), p. 192. The concept of material adequacy is somewhat more difficult to define. Part of it is that the deontic

intuitions produced by the knowledge of modal reality”⁹. Arthur Prior, e.g., derives from von Wright’s system the “paradox of derived obligation”:

the doing of what is forbidden commits us to the doing of anything whatsoever. (Stealing, e.g., —supposing that to be forbidden—commits us to committing adultery; and also, of course, to not committing adultery)¹⁰.

In von Wright’s terminology, “A commits us to B” is in his symbolism “ $O(A \rightarrow B)$ ”, where “ $O(\dots)$ ” is the deontic operator of obligation and “A” and “B” stand for names of act(ion)s, such as “theft” (in general, not any one particular theft)¹¹. But if A is theft and B is adultery, the von Wrightian commitment does not mean that “when we have stolen we thereafter [and for that reason] stand under an obligation to commit adultery”¹²—which is what our “naive”, “pre-logical” or “real-life” concept of commitment implies. No, the von Wrightian commitment only means that “stealing accompanied by refraining from adultery [or anything else] is still stealing, and so is still wrong; and it means no more than that”¹³. Prior opines, however, that it *ought to* mean more¹⁴. To what purpose? Conceivably, to make the von Wrightian concept of commitment more adequate for the task of regimenting ours, the one that we use to talk about the diverse *ought to*’s and *mustn’t*s and other deontic modalities of this real life of ours.

Not being a logician, I shall not try to propose a method of blocking the paradoxes and “devastating counter-examples” by making the formalism more sophisticated. Yet, my concern being the applicability of deontic logic to

operators, such as “ $O(\dots)$ ” (it is obligatory...) or “ $P(\dots)$ ” (it is permitted...) and the others should harmonise with the use of the “real-life” deontic expressions they are supposed to formalise, in this case “it is obligatory...”, “it is permitted...” and the like.

⁹ G. Kalinowski, “Norms and Logic”, *cit.*, p. 192.

¹⁰ A.N. Prior, “The Paradoxes of Derived Obligation”, in *Mind*, 1954, 63(249), p. 64.

¹¹ I take the reader’s familiarity with sentential calculus, its connectives (such as “ \sim ”, “ $\&$ ”, “ \rightarrow ”, “ \vee ”) and its basic laws (such as De Morgan, Commutativity, Distributivity and the like) for granted.

¹² *Ibidem*.

¹³ This is Prior’s “prose” for von Wright’s $O(\sim p) \rightarrow O(\sim(p \& q))$, against which yet another remark may be made: what if q is an act of expiation and/or redressing and reparation? Are we sure that even then p & q is still forbidden? Even if the compensation exceeds the wrong done, as, e.g., in *Job* 42, 10–17? If p is forbidden and is in no way connected with q (nor is q with p), e.g., p=theft, q=donating to charity (but not the goods or money stolen) it makes sense (so at least is the present author’s “intuition”) to say “no matter what else you do, it’s wrong to steal”. But if the theft and the philanthropy *are* connected, say *à la* Robin Hood, an attempt at arguing that q is in a way “redeemed” by q need not be doomed to failure. Still less, I believe, need it be so if q is compensation for damage or theft. In fact, cases are known in which Peter commits a petty theft on Paul with a view to him (or somebody else) recompensing Paul afterwards beyond the nominal worth of the property stolen; the motivation being the desire to raise Paul’s “quality of life” without going into the trouble of persuading Paul to accept a gift.

¹⁴ *Ibidem*.

“real-life” deontic discourse¹⁵, legal, ethical, prudential, etiquette (“good manners”), “epistemic deontics¹⁶” etc., I shall offer some observations of how in my view not sufficiently carefully constructed “real-life” examples of deontic sentences render what goes by the name of Standard (Monadic) Deontic Logic¹⁷ (SDL) more “bordering on the nonsensical” than it probably is. Unlike some scholars (e.g., Perelman¹⁸, or Amselek), I do believe in the possibility of controlling the correctness of deontic reasonings by means of a deontic logic. I do *not* believe that the principles of deontic logic cannot help “bordering on the nonsensical”. In my opinion, the “nonsensicality” is often due to the inattentiveness with which examples are constructed and pressed into the (actually or only supposedly) “Procrustean bed” of the formulae of SDL.

2. The problem(s) with the K-axiom

Actually, I shall limit myself to just one “Procrustean bed”, namely, the all-important Distributivity Axiom, a.k.a. the K-axiom, of SDL, which reads:

$$O(p \rightarrow q) \rightarrow (O(p) \rightarrow O(q))$$

where $O(\dots)$ is the operator of deontic necessity (in common English “one ought to ...”, “you must...”, “they are obliged/obligated/under the obligation to...” and the like, too many to list them all here, while “p”, “q” and the like are propositional variables, the propositions being about actions and omissions— or sometimes about sentences expressing such propositions, also names of actions and omissions specified in these sentences and/or propositions, states of affairs and others; this ambivalence is perhaps harmless for a proficient logician but may be found confusing by a “real-life” practising “deontician”).

The K-axiom is essentially involved in the derivation of many, maybe most, paradoxes of deontic logic. My observations below will pertain to just a handful of such paradoxes” some of them set out by McNamara and Van De Putte¹⁹.

¹⁵ For aught I understand, this is a concern motivating, among many other things, the book by J.W. Forrester, *Being Good and Being Logical: Philosophical Groundwork for a New Deontic Logic*, M.E. Sharpe, Armonk/London, 1996. Quite a few scholars of highest renown have expressed a similar concern in the past, e.g., O. Weinberger, “Normenlogik anwendbar im Recht”, *cit.*

¹⁶ See “The Normativity of Logic”, in <https://www.uib.no/en/antiex/99273/normativity-logic> [accessed 20/12/2022].

¹⁷ As codified by P. McNamara, F. Van De Putte, “Deontic Logic”, *cit.*

¹⁸ C. Perelman, “Droit et morale”, in *Akten des XIV. Internationalen Kongresses für Philosophie, Wien, 2. 9. September 1968*, vol. 4, Herder, Wien, 1968, pp. 269–311; C. Perelman, “Interventions au colloque de Bruxelles”, in C. Perelman (ed.), *Études de logique juridique. Le raisonnement juridique et la logique déontique: actes du Colloque de Bruxelles (22–23 décembre 1969)*, Émile Bruylant, Bruxelles, 1970.

¹⁹ P. McNamara, F. Van De Putte, “Deontic Logic”, *cit.*, s. 2, ss. 4.1.

Part of the problem is that the K-axiom is *not* exactly famous for its wide and uncontroversial applicability to real-life deontic discourse. Von Wright held it for a truth of logic²⁰, yet its status as a real-life deontic propositions of the form “ $O(p \rightarrow q)$ ” (the protasis of the K-axiom)—expressing *unconditional obligations with conditional content*, as I shall occasionally be calling it, following Reinach²¹—is dubious. Kalinowski holds that such sentences do not occur at all in “real-life” deontic texts²². Weinberger claims that “‘ $O(p \rightarrow q)$ ’ *kommt [...] als Strukturschema des Bedingungsnormsatzes überhaupt nicht in Frage*” for structural reasons which he then specifies²³.

More generally, there are, in SDL, problems with the sentences in which deontic operators host complex formulas within their scope. Is, for instance, $O(p \vee q)$ equivalent to $O(p) \vee O(q)$ ²⁴? What *is*, in general, the “disjunctive action” “ $p \vee q$ ”? Is, e.g., someone who is posting a letter—posting a letter or at the same time declaring war on Ruritania, or posting a letter or dropping the “Fat Man” on Nagasaki? The latter I find weird (just I?), the former—too, *unless*, perhaps, the letter contains precisely a formal declaration of war on Ruritania. What, then, about posting a letter *or* burning it? Can I discharge my duty to post the letter by burning it, under the pretence that if $O(p)$ then $O(p \vee q)$ ²⁵ then $O(p) \vee O(q)$ and I am in the habit of fulfilling my duties starting from the last ones in the order of derivation²⁶? Does it make any difference that the “disjunctive action” (Weinberger’s *Handlungskomplex*²⁷) of posting the letter and dropping the bomb consists of two completely unconnected actions, whereas mailing the letter and declaring war *may* be, effectively, one and the same action under two distinct descriptions, while the actions of burning the letter or posting it are mutually

²⁰ G.H. von Wright, “Deontic Logic”, *cit.*, p. 5: “It is intuitively obvious that this is a truth of logic, i.e. something which is valid on purely formal grounds. It is, however, not an application of any scheme which is valid for any sentences, whether deontic or not. The existence of logical truths which are peculiar to deontic concepts is what makes the study of Deontic Logic interesting”, but also tantalising and vexing to a practising “deontician”.

²¹ A. Reinach, “The A Priori Foundations of Civil Law”, in *Aletheia*, 1983, 3, p. 23 and cf. p. 27; Żelaniec, Wojciech, 2017), “The Challenge of the K-Principle in Deontic Logic (and Well Beyond)”, *Phenomenology and Mind*, (13), pp. 138-149. doi: 10.13128/Phe_Mi-22436. p. 142. A reader who has made it thus far in the present paper may be interested in this former piece of mine as well as some of the papers quoted there as references.

²² G. Kalinowski, *Einführung in die Normenlogik*, Athenäum, Frankfurt, p. 56; G. Kalinowski, “Sur les sémantiques des mondes possibles pour les systèmes de logique déontique”, in *Logique et Analyse*, 1981, 24(93), pp. 81 and 87f.

²³ O. Weinberger, “Erwiderung auf A. Soetemans Kritik und seine Reformversuche der deontischen Logik”, in *Logique et Analyse*, 1973, 16(61/62), p. 288.

²⁴ G.H. von Wright, “Deontic Logic”, *cit.*, p. 6f.

²⁵ “If $\vdash p \rightarrow q$ then $\vdash O(p) \rightarrow O(q)$ ” is a rule of derivation in SDL, see P. McNamara, F. Van De Putte, “Deontic Logic”, *cit.*, s. 2.1.

²⁶ This is the so-called Ross Paradox. See P. McNamara, F. Van De Putte, “Deontic Logic”, *cit.*, s. 6.3.

²⁷ O. Weinberger, “Erwiderung auf A. Soetemans Kritik und seine Reformversuche der deontischen Logik”, *cit.*, p. 286.

exclusive? In which of these cases can we speak of just one, albeit disjunctive, action for a deontic operator to operate on? Analogous problems arise for $O(p \ \& \ q)$ and $O(p \rightarrow q)$ ²⁸. Or is $\sim p$ simply not doing (omitting) p or is it *refraining* from p 'ing²⁹? Yet more generally: how do logical connectives operate on (names) of (types) act(ion)s? Or maybe, *pace* von Wright, the variables range over propositions, not over acts or types of acts? What is, e.g., "it" in "it ought to be that ..."? A sentence/proposition, "be" meaning then "be true"? Or a state of affairs, whatever states of affairs are, "be" meaning then "obtain", "hold" or whatever is the appropriate kind of being for states of affairs? A practising "deontician", eager to learn and apply deontic logic cannot sometimes help feeling that professional deontic logicians are not always very scrupulous in distinguishing between use and mention or between various levels of language—probably because they fill in such details "on the fly" while he, alas, does not.

Returning to the K-axiom: A few examples, or putative examples (instantiations, particular cases) thereof *have* been provided. And many of them make problems with it come into sight almost immediately.

One of them is: "It is obligatory, if you walk in a public place, to be wearing clothes"³⁰. OK, it does not sound all that "real-life", so let me start from one of my own making, which I believe is somewhat pedestrian:

2.1. Money (to be) borrowed

It is obligatory, if you borrow money, to return the money in due time and with due interest.

Formalising it as " $O(p \rightarrow q)$ " instead of " $p \rightarrow O(q)$ "³¹ is, I think, better for "educational" reasons: it is better to generate in the recipient of such norms the impression that he is under an obligation *from now on*, e.g., from the moment a norm like that has been proclaimed valid and not only *could* be under an obligation after he has performed some other act, which he might not at all be likely to ever perform³².

²⁸ O. Weinberger, "Normenlogik anwendbar im Recht", *cit.*, *passim*; O. Weinberger, "Erwiderung auf A. Soetemans Kritik und seine Reformversuche der deontischen Logik", *cit.*, p. 286f. and *passim*.

²⁹ See, among many others, e.g., D.N. Walton, "Omitting, Refraining and Letting Happen," in *American Philosophical Quarterly*, 1980, 17(4), pp. 319–26.

³⁰ R.N. McLaughlin, "Further Problems of Derived Obligation", in *Mind*, 1955, 64(255), p. 400.

³¹ A form not well-formed in G.H. von Wright, "Deontic Logic", *cit.*, which admits only formulae with (negated) deontic operators as arguments of uppermost level logical connectives. It is not clear to me whether this form is legal in SDL; in any event, it *can* be occasionally found in P. McNamara, F. Van De Putte, "Deontic Logic", *cit.*

³² The downside of this approach is that we are bound to see ourselves as entangled in a plethora of absolute obligations with conditional content whose protases might at times be rather weird, such as the obligation to, if we should be murdering our maternal grandfather, do it gently and

This seems to be quite a “real-life” situation: We are always and eternally obliged—though probably only *prima facie*—to return borrowed money (“ $O(p \rightarrow q)$ ”), and in addition, for some special reasons—e.g., because we ought to provide for our family and at the moment we have no other lawful ways of coming by the necessary funds—we are obliged to borrow (an amount of) money ($O(p)$). Are we then obliged to return the money in due time and with due interest—which is what the K-axiom obligates us to deduce?

The difficulty is that the money may not have been borrowed at all. We may have defaulted, in a variety of causes and reasons, both within and outside our control, on our obligation to *borrow* the money in the first place. Assuming that no such reasons have cancelled this obligation, are we still under the other obligation, that of paying back the money? There are two answers possible: “No, we aren’t. There is nothing to pay back if we haven’t borrowed anything and consequently there can be no obligation to pay back that non-existent amount.” (A) “Yes, we are, and this obligation can be discharged easily, namely by doing nothing”³³. Answer B saves the reputation of the K-axiom hands down, if at the price of introducing a rather peculiar kind of obligations; answer A saves it too, in that it shows that in the situation in hand the K-axiom simply does not apply as $O(q)$ is not well-defined.

However, someone could argue that $O(q)$ is defined, and is false, since it is not possible to pay back non-existent money and *impossibilium nulla obligatio*³⁴. Presupposing $O(p \rightarrow q)$ and $O(p)$, this instantiation of the K-axiom comes out false³⁵.

To forestall this objection, it is also possible to put the matter thus: there are in fact two distinct “monies” involved here. In $O(p)$ it is *pecunia mutuo sumenda*, as Cicero would say, while in $O(q)$ (as standardly understood) it is *pecunia mutuo sumpta*, i.e., money to-be-borrowed and money actually borrowed. English does not make this distinction that Latin does, viz., between the future passive participle with an “oughty” connotation and the past passive participle restricted

humanely, and so on. Besides, this awareness of being under an obligation from the present moment onwards is somewhat illusory, as we shall see later on.

³³ If the presence at a faculty meeting was obligatory the meeting’s point was to process the applications submitted up to yesterday, but there are no such applications, we can congratulate ourselves on having done a good job and leave immediately. Perhaps a special ritual could be invented for duly “returning” never-borrowed money if they ought to have been borrowed by whomever is now “returning” them going through this ritual. Such rituals may be useful whenever the non-borrowing was contrary to (a) duty.

³⁴ G. Feis, *Impossibilità nel diritto*, ETS, Pisa, 2015.

³⁵ To this author, at least, it appears dubious if the principle *impossibilium nulla obligatio* is really applicable to cases like the present one, i.e., when it an action is impossible not because it is beyond the powers and means of the agent but because the object of the action does not exist. Dethroning Charles III is really beyond me, so I have no obligation to do it, but is it true in the *same* sense of “having no obligation” in which I have no obligation to dethrone the present king of France? This author feels under no obligation to decide this question in this essay and he is not even up to the task to decide if this is due to the problem’s being simply too difficult for him or to there not being any such difference and no critical “matter of (deontic) fact”.

to describing accomplished things. In “O(p)” the action is on the first *pecunia*, in “O(p)” by contrast it is an action on the second one—which, however, need not to exist. But in “O(p → q)”, on its most natural (as I think) reading, there are no *two actions on two different objects* but just one: returning (action) borrowed money (object)³⁶. Maybe this difference could and should be somehow reflected in the symbolism, e.g., $O(p \rightarrow q) \rightarrow (O(r) \rightarrow O(s))$?

2.2. Chisholm Puzzle/Contrary-to-Duty Paradox

A somewhat similar, but not identical, problem with the K-axiom is to be found in the context of the so-called Chisholm Puzzle³⁷, also known as the Contrary-to-Duty Paradox. Chisholm accepts, for the sake of argument, both the K-axiom and the formula “O(A) → ~O(~A)”, which is, indeed, an axiom of SDL. His “O(p → q)” reads: it is obligatory that if Jones goes to his neighbours to help them out with something he should tell them that he will be coming” and his “O(p)” reads: “it is obligatory that Jones should go to his neighbours and help them.” From this, it follows by the K-axiom that Jones is obligated to inform his neighbours that he will be coming (O(q)). There is yet another premiss: that if Jones is not coming to help his neighbours, he is under the obligation *not* to tell them that he will come. ($\sim p \rightarrow O(\sim q)$)³⁸. And then, John decides, or is made, not to come: $\sim p$. As a result: O(~q). Yet, O(q); and O(A) → ~O(~A). Inconsistency.

It is not a strikingly “real-life” example. In a situation like that most people (at least in the “Western” world), should they no more be willing or in the position to come and help their neighbours, would either think they are obliged to tell their neighbours that, contrarily to the former arrangement, they won’t come, or—they would simply do nothing. To tell them one *will* be coming despite having no (more) intention to do so—who but the most mischievous would do such a thing³⁹?

But how do we conceptualise the “telling the neighbours that we are coming” part? In other words, what does “q” here stand for? Is it just saying the words, e.g., “I am coming to your assistance, neighbour!” or the like? If so, then the K-axiom is again defied. However, we can, or perhaps rather should (epistemologically), conceptualise the “telling the neighbours that we are coming”

³⁶ This is my modest proposal of how to understand “A → B”, where the variables range over act(ion)s, in other words, of how to understand the concept of “implicational action” as *one* action—at least in the “financial” context presupposed here.

³⁷ R.M. Chisholm, “Contrary to Duty Imperatives and Deontic Logic”, in *Analysis*, 1963, 24(2), p. 34f.; P. McNamara, F. Van De Putte, “Deontic Logic”, *cit.*, s. 4.1.

³⁸ See note 31.

³⁹ Such a “prank” seems to make even less pragmatic sense than that of the sailors from *Dido and Aeneas*, Act III, Prelude, who sing: “Take a bouzy short leave of your nymphs on the shore, / And silence their mourning / With vows of returning, / But never intending to visit them more.” Maybe Chisholm had a fairly unusual set of circumstances in mind while inventing his example—but failed to/refrained from/decided not to/forgot to make them explicit.

as telling a falsity in $\sim p \rightarrow O(\sim q)$ and telling a (hypothetical) truth in the other premises. Telling a truth and telling a falsity being two rather different things “ $\sim p \rightarrow O(\sim q)$ ” should be rewritten as, e.g., “ $\sim p \rightarrow O(\sim r)$ ” and the inconsistency will be removed.

2.3. McLaughlin’s dressed walker

Now let us examine McLaughlin’s example: “It is obligatory, if you walk in a public place, to be wearing clothes”. As distinct from returning money one has not at all borrowed, it is very well possible to wear clothes even if *not* walking in a public place. So, if there is the absolute obligation with conditional content, viz., “you ought to, if walking in public places, be wearing clothes” and if there is, in addition, an obligation to walk in a public place, the K-axiom seems to be saying that there, then, is an obligation to wear clothes, and that even if we are, contrarily to the latter duty (to walk in public places) *not* walking in any public place. This sounds implausible. The K-axiom defeated? On reflection, we see that the absolute obligation with conditional content of wearing clothes while walking in public places deserves a more careful wording. What kind of action⁴⁰ does it effectively oblige us to⁴¹? Not to suddenly transforming ourselves from naked to clothed the very moment we cross the boundary between a private and a public place; that is sheer magic. Given the structure and the usual properties of human agency, it requires, much rather, that on anticipating walking in a public place (for whatever reasons) we should first see to it that we have on some clothes. At the same time, the given obligation to walk in a public place is just that: to walk, not to *anticipate* walking, in a public place. Thus, we end up with the following:

if it is obligatory to put on some clothes while anticipating walking in a public place, before leaving home then, if it is obligatory to walk in a public place, then it is obligatory to have some clothes on, which is $O(r \rightarrow s) \rightarrow (O(p) \rightarrow O(q))$, rather than the K-axiom.

Again, a counter-example has been disarmed.

⁴⁰ Note that “wearing clothes”, “being dressed/clad”, “having clothes on” or however else you should like to word it is not an act(ion). Here the somewhat lax style of determining what exactly can be obligatory, permitted and forbidden is calling in its due (see note 7). However, we can see such states of affairs (if such they be) as results of actions, such as putting on clothes or having someone else do it for us.

⁴¹ This type of question merits being asked more often by every student of norms interested in their role in “real life”. Norms are typically issued with a view for influencing human behaviour, and their proper understanding requires taking into account this function, including their “naive” understanding by their addressees.

2.4. Ziemba's ticket buyer

McLaughlin's is an outlandish example, so let's examine this one by Ziemba: the act of going anywhere by train "obliges me also to buy a railway ticket"⁴². Now suppose it is obligatory for me to make a train trip to Warsaw. But what if I won't? Am I still obliged to buy a ticket? On closer inspection we see that, at least in a typical case, the protasis of this instance of the K-axiom is something like:

It is obligatory, if you anticipate a (sufficiently determinate, as regards route, date etc.) train trip, to buy a respective ticket.

while the protasis of the apodosis of this instance of the K-axiom is simply "making a train trip", not "anticipating one". This is representable as $O(r \rightarrow q) \rightarrow (O(p) \rightarrow O(q))$, rather than as the K-axiom. Again—a counterexample to the K-axiom has been disarmed⁴³.

Moreover, you may ask what would happen if there actually were an obligation to *anticipate*, either a train trip, or walking in public places? Would the respective duty to buy a ticket or to put on (rather than have on) some clothes not be there as well? If yes, the K-axiom would be borne out and the counterexample nipped in the bud. I am indeed inclined to answer this question with a "yes"; yet, the example is too eccentric to have any firm intuitions about. I think, however, that many of us would recognise analogous, and more intuitive, cases. For instance, many of us consider themselves under the (prudential) obligation to anticipate various circumstances of human life, such as old age, illness or death, and some of us think themselves under the standing obligation to, if they anticipate such necessities, actually provide for them; as a result some of us also feel under the obligation to actually provide for these to-be-expected circumstances. But I suspect that the reasoning involved here is heavily enthymematic⁴⁴, and does not rely on the K-axiom alone, but also on additional premises, which I am hard put to make out, and, in any event, on the logical links between anticipating and that which anticipating essentially consists in. (An inseparable part of *my* anticipating such events as concern *me* is *my providing for them*).

⁴² Z. Ziemba, "Deontic Syllogistics", in *Studia Logica*, 1971, 28, p. 155.

⁴³ In a similar fashion one could, I suppose, deal with the other example of McLaughlin's, that of taking a seat in a smoking compartment while not smoking. See R.N. McLaughlin, "Further Problems...", *cit.*, p. 400.

⁴⁴ As is, I suspect, Forrester's example (J.W. Forrester, *op. cit.*, p. 30): "Smith ought to concede the election, Smith ought to, if she concedes the election, hold a gracious speech, then Smith ought to hold a gracious speech". She ought to hold a gracious speech because, given that she (knows she) ought to concede the election, she must have realised that she has lost the election and if she doesn't protest she has virtually conceded it, hence... Or maybe: conceding the election and holding a gracious speech is the same act? Something like this.

3. A clear counter example for the K-axiom?

Here is an example which seems to fit the K-axiom well⁴⁵ and not to contain anything fishy, and seems to be close to our (real, if academic) lives:

I promise my family to spend the next week entirely with them if I manage to finish writing this paper till Sunday.

Note well that this self-imposed obligation is valid from the very moment the promise is made onwards, so there is hardly any doubt that it has the requisite logical form, that of the protasis of the K-axiom: “ $O(p \rightarrow q)$.” It is clearly *not* a “conditional obligation with unconditional content⁴⁶” of the form $p \rightarrow O(q)$, as the obligations comes into being instantly and does not wait till something else has come into being⁴⁷. Then suppose that I have, in addition, an independent obligation to actually finish this paper till Sunday ($O(p)$). Do I, then, by the very self-same token have the obligation to spend the next week with my family? I think common sense would say “no”. If the answer were “yes”, I could have just as well promised my family to spend the next week with them, full stop, and no “ifs” prefixed. As things stand now, however, they can at most hope that I discharge the obligation of getting my paper ready by Sunday. If this is correct, then we have found a clear counterexample to the K-axiom. $O(p \rightarrow q)$, $O(p)$, and yet $\sim O(q)$. How come?

It is impossible that it should come, if we rely on the (standard understanding of the) standard Kripkean possible-worlds semantics⁴⁸. In it, as we know, $O(\alpha)$ is true, in a given world w , iff α is true in all the worlds that are possible and deontically accessible, or “ideal”, or “acceptable” or “admissible”, from world w . And which worlds are accessible in this sense from a given world⁴⁹? The main idea, as far as I could glean from the relevant literature, seems to be that a world is “deontically accessible” from this one iff all obligations valid in this world are duly discharged in that other world. On this basis, there simply *cannot* any counterexample to the K-axiom. (More on this in the second paragraph of § 5 below.)

But we feel, if we try to be honest to ourselves (and to our family), that $\sim O(q)$, all the same. Once again: how can this be?

⁴⁵ I.e. it seems to have the requisite logical form: $O(p \rightarrow q) \rightarrow (O(p) \rightarrow O(q))$.

⁴⁶ A. Reinach, *op.cit.*, pp. 23–27.

⁴⁷ This is at least how my family interpret my promise...

⁴⁸ P. McNamara, F. Van De Putte, “Deontic Logic”, *cit.*, s. 2.3.

⁴⁹ Cf. J. Woleński, “Deontic Sentences, Possible Worlds and Norms”, in *Reports on Philosophy*, 1982, 6, pp. 66–73. New corrected version as: J. Woleński, “Deontic Sentences, Possible Worlds and Norms”, in *Revus. Journal for Constitutional Theory and Philosophy of Law/Revija Za Ustavno Teorijo in Filozofijo Prava*, 2018, 34.

Part of the solution might be, I should like to submit, realising that, when we read, hear or say something like $O(p \rightarrow q) \rightarrow (O(p) \rightarrow O(q))$ we ask ourselves, in order to make sense of this somewhat weird (see note 22 and 23) protasis “ $O(p \rightarrow q)$ ”, what it *really* commits us to, we arrive at something like “to not: to p and yet not to q”. Whereupon we *sometimes* imagine p as already done and we focus on the injunction not to p (but p, we assume, has already been done), and leave q undone. Taking the achievement of p for granted, we place ourselves in the shoes of successful ourselves⁵⁰, who have accomplished p and ... we inevitably end up imagining that $O(q)$.

But this is an illusion. No matter how suggestive the image of p successfully done is, we are not yet *there*, i.e. at the place where it starts being true that $O(q)$. A bride(groom) during the wedding ceremony, separated by just a few words from her/his “I do” may well think: “Now I am responsible for him/her and (s)he for me”, but (s)he isn’t yet. “There’s many a slip ’twixt the cup and the lip” and things may happen to prevent her/his “I do” from being solemnly pronounced, and even after both parties’ “I do” has been solemnly pronounced, other things may transpire that will keep one or both of the parties from assuming all the diverse duties and rights legally connected with the institution of marriage.

Why do we fall for this illusion? I think part of the explanation is that we take “ $O(q)$ ” too seriously, and this for a variety of reasons. Contracting a marriage, getting a paper ready, being a flâneur for an hour or two are all agreeable kinds of experience, so we are eager to image we are on that stage already, especially if duties generated thereby are, too, pleasant (being a flâneur and showing off one’s new clothes, say). In other cases, the inevitability of a certain stage (age-related infirmity) makes the prudence of anticipating it an option without alternatives, even if we haven’t yet started practising this prudence...

In yet other cases, it may be sheer sense of duty that has us imagine we have already satisfied requirements. In replying to McLaughlin’s wearing-clothes-while-not-walking-in-public-places example, von Wright says⁵¹: yes, it is obligatory to wear clothes even when one is not walking in public places

and [...] the premisses of the problem being taken into due consideration, this is in good accord with our intuitions. For, if walking in a public place really is obligatory, then to neglect this action is forbidden, and the only way in which we are allowed to fulfil the obligation imposed in the second premiss (viz., not to walk in a public place or be dressed) is by being dressed. So that here the two acts of walking in a public place and wearing clothes are both obligatory.

⁵⁰ No doubt a bit of wishful thinking is at work here as well.

⁵¹ G.H. von Wright, “A Note on Deontic Logic and Derived Obligation”, *cit.*, p. 507.

But we can well not at all be impressed with what “really is obligatory” and not give a damn for what consequently is “the only way in which we are allowed to fulfil the obligation imposed in the second premiss”.

Moreover, we can even be negatively impressed by “O(p).” According to a legend, Spartan soldiers heading off for war were advised to return either *with* their shields or *on* their shields: Fleeing soldiers usually dropped their shields, while dead soldiers were carried on their shields by their companions. This injunction could be symbolised thus: $O(p(=you\ don't\ return\ with\ your\ shields) \rightarrow q(=you\ return\ on\ them))$. Typically, soldiers are sent off to the battlefield with the exhortation to win and not to be routed and killed; but exceptionally the commander-in-chief could wish most of his soldiers to actually honourably die on the battlefield, so his second command would be “don’t return with your shields”. Would these soldiers eagerly start making provisions for being carried⁵² back home on their respective shields? Probably with less eagerness than I start looking forward to my week with my family, even though I haven’t finished my paper yet.

To prevent this kind of illusion from arising it may be helpful to realise that “p” and “q” in “O(p → q)” stand for actions⁵³ (as) merely contemplated and free from any deontic characteristics, whereas in “O(p)” and “O(q)” they stand for “the same” actions in the ought-mode (so that in fact different variables should be employed for them) as actions to be done. (And we have seen that such a modus cannot be meaningfully assigned to actions whose objects have not yet achieved the is-modus: returning borrowed money cannot be obligatory if it still to-be-borrowed money. But certainly, a person getting ready to make to-be-borrowed money borrowed money may, understandably, feel already under the obligation to return “it”, even though, technically, that obligation does not yet exist, has not arisen. And similarly, a person very close to finishing his paper by Sunday midnight may well already feel under the pleasant duty to spend the next week with his family.

4. ... and an equally convincing pro example

The same kind of psychologising can be brought to bear on a reasoning in which seems to be a perfect example of the K-axiom⁵⁴ and which James Forrester thinks

⁵² If *all* of them anticipated being killed, the provisions would include getting someone else to carry them back to Sparta on their shields, not an easy matter.

⁵³ Or components of an action (see note 29). Logicians (including the great Weinberger, see note 23) are not pedantic with regard to the status of creations of this form: [name of action] [logical connective] [name of action]. Are they, too, actions? Or are they complexes of actions? If the former, how can their component actions be combined? If the latter, how can logical connectives and deontic operators operate on them?

⁵⁴ Not just in the sense of having its logical form but also in that of providing a convincing example of its truth.

every deontic logic must legitimise as valid⁵⁵: If Tamino is obliged to enter a temple through one of three doors (that is, he is obliged, if he doesn't enter it through either the rightmost or the leftmost, to enter it through the middle, $O(\sim p \rightarrow q)$, but also, he is actually obliged to not enter it through the leftmost and the rightmost one ($O\sim p$); hence he is—is he not?—obliged to enter it through the middle door ($O(q)$). This reasoning seems immediately convincing, just as much as does the former, that with the paper-to-be-ready-by-Sunday, only to the contrary effect; why? In a nutshell, I think, it is because when Tamino is confronted with these two former obligations ($O(\sim p \rightarrow q)$, $O\sim p$), he immediately realises that he has discharged the second obligation, $O\sim p$, just by doing nothing. “How can I then discharge the first obligation?” he is asking himself, and seeing that he has made $\sim p$ true and thereby discharged $O\sim p$ (as he has not yet entered the temple through any of the three doors), he realises that he can't discharge it except by doing q , which is why he feels that $O(q)$. This is what $O(\sim p \rightarrow q)$ “boils down” for him to. This psychology would suggest, in lieu of the K-axiom, a principle like $O(p \rightarrow q) \rightarrow (p \rightarrow O(q))$, which, however, probably does not get a deontic logician very far. Unlike in my example with the paper due by Sunday, there is nothing, in Tamino's case, to wait and see if it really happens, and no risk of exposing oneself to things turning out the wrong way (“gosh, it's Monday midnight and I haven't finished my paper yet...”), nothing to—yet—trigger off the obligation to q . If Tamino had been obliged, not: to enter the temple but through neither the leftmost nor the rightmost door, but, rather, to think about it for a minute and decide on his own against entering the temple through either, Tamino would have to wait and see if he really decides against that before feeling obliged to enter it through the middle door.

5. Can there be deontically imperfect yet deontically “accessible” worlds?

This author is somewhat reserved with regard to “possible worlds” and with all due respect for the Kripke semantics he is unable to see in them much more than a (sometimes) useful thought-model, and still less an unrevisable one. One small revision, inspired by the above considerations, that he should like to propose is this.

A world, it seems to me, can be deontically (but no more than deontically) as “accessible” as ever, even if *not all* obligations valid in ours are fulfilled in it. This is seldom, if ever, considered, let alone admitted as a possibility, by students of the matter; Woleński states without a qualification: “Clearly, in deontic alternatives [i.e. deontically accessible possible worlds] no prohibited action can be realized⁵⁶”. Certainly, I can fail to be done with my paper by Sunday because I

⁵⁵ J.W. Forrester, *op. cit.*, p. 17.

⁵⁶ J. Woleński, “Deontic Sentences, Possible Worlds and Norms”, *cit.* New corrected version as: J. Woleński, “Deontic Sentences, Possible Worlds and Norms”, *cit.*

am lazy, but also, because I am too honest to gloss over some basic problem I have just found in my paper and I cannot summon up any more wits to solve it by Sunday. Another reason may be a power outage due to a lightning. A deontically—but no more than deontically—acceptable, though perhaps less than “perfect” purely and simply, world should provide for all kinds of *vires majores*—natural catastrophes, epidemics, brain strokes, fits of dullness and such like—that make the fulfilment of some of its obligations impossible even to the most eager and most conscientious. Such *vires majores* do not always literally annihilate the obligations whose discharging they make impossible (the so-called Kant’s⁵⁷ Law does not always apply in its literal sense if the obligations are “in principle”, i.e. abstraction made from adventitious circumstances, fulfillable), but may, and in some cases do, perfectly *excuse* not acting on them. There is, it is true, a notorious tendency of human nature to look for the culprit whenever something goes wrong, but it should be kept in check. So I think it is better to think of deontically acceptable (from the viewpoint of ours) worlds as those in which—amongst other things—all valid obligations of our world are either fulfilled or excusably unfulfilled.

And there is more: Sven Ove Hansson is unhappy (not with possible worlds as such but rather) with the way we use possible worlds in deontic logic⁵⁸, and so am I. He observes, for instance, that in a deontically perfect world nobody would be in the position to discharge a number of important obligations, such as that of opposing injustice⁵⁹. There *is* no injustice in such worlds, Hansson thinks, and hence nothing to bravely oppose. So such worlds are in a sense “too perfect”. This perhaps could be answered by proposing that the obligation to oppose injustice has, in fact, the logical form of “ $p \rightarrow O(q)$ ”: “on encountering injustice, it is obligatory to oppose it”, rather than “it is obligatory, on encountering injustice, to oppose it” and if there is no injustice then this obligation can be “discharged vacuously” by simply doing nothing. But I think that even in a deontically admissible world, as I’d like to conceive of it, there can be *excused* injustice.

Petty excused injustice may be more frequent than you would think; for instance, excused by time pressure or similar constraints on resources (think of the notes to give to your students and ask yourself: are they always all that just? And

⁵⁷ Not really Kant’s: “*impossibilium nulla obligatio est*” (there is no obligation to impossible actions), formulated by the Roman lawyer Publius Iuventius Celsus in the 2nd century Christian era in Rome, and made part of Justinian’s *Digesta* (50.17.185). See G. Feis, *op. cit.*; M.J. Schermaier, “*Impossibilium nulla obligatio. Vorverständnis, Begriff und Gegenstand der Unmöglichkeit der Leistung im römischen Recht*”, in *Annali del Seminario giuridico dell’Università di Palermo*, 51, 2006, pp. 241–268.

⁵⁸ S.O. Hansson, “Ideal Worlds: Wishful Thinking in Deontic Logic”, in *Studia Logica*, 2006, 82(3), pp. 330–332.

⁵⁹ Cf. the utopia sketched by Hume in his *Enquiry concerning the Principles of Morals*, section III, part I, pp. 183f. in SBN: “Let us suppose, that nature has bestowed on the human race such profuse abundance of all external conveniencies [...] It seems evident, that, in such a happy state [...] the cautious, jealous virtue of justice would never once have been dreamed of. For what purpose make a partition of goods, where every one has already more than enough?”

yet, you know that you are always doing your best so that they should be just...). And there is also a lot of not-so-petty, rather: immense excusable injustice, denounced by the social teaching of the Church as “structures of sin”: large scale social structures which leave us with no reasonable choice but to do acts which taken cumulatively add up to injustice despite nobody’s ever willing them to do, or even acquiescing in doing, injustice⁶⁰. There are, too, cases of medium-range injustice, as in the case of a German called Khaled El-Masri, who was given the benefits of an “extraordinary rendition programme” by the CIA until it turned out that he was the wrong guy, but the Supreme Court refused to hear his case and thereby excused the injustice done to him⁶¹... But this is getting far too “real-life” already.

6. The K-axiom immunised?

But can all these aspersions cast on the K-axiom not be once and for all dispelled by the reminder that von Wright has proven it a tautology of his system of 1951⁶², in fact the *ur*-system of “every future SDL that would be able to present itself as a solid and useful system of formal logic” and not as a collection of paradoxes “bordering on the nonsensical”? Certainly. The matter is only to make von Wright’s proof doubt-proof. And this is not so easy to do, if you can anticipate all the quirks of your typical doubt-raiser. Now the final step of that proof is the reduction of the K-axiom to a tautology of sentential calculus, viz., $p \& q \& r \rightarrow r \& q$. The procedure does not presuppose any knowledge of logic beyond classical sentential calculus plus the definition of “P(...)” as “ $\sim O(\sim \dots)$ ” (equivalent to “O(...)” defined as “ $\sim P(\sim \dots)$ ”). This definition enables von Wright to rewrite the K-axiom (in the form of “ $O(A) \& O(A \rightarrow B) \rightarrow O(B)$ ”⁶³) as “ $\sim P(\sim A) \& \sim P(A \& \sim B) \rightarrow \sim P(\sim B)$ ”. The first critical move comes next and consists in replacing the “arguments” of the three “ $\sim P(\dots)$ ”s with their disjunctive normal forms. These forms are, as it were, exhaustive descriptions of which elementary acts may have been done in the given situation (in which only these acts are taken into consideration); for instance, the disjunctive normal form of “ $\sim A$ ” looks like this: “ $\sim A \& B \vee \sim A \& \sim B$ ”. The result is:

$$\sim P(\sim A \& B \vee \sim A \& \sim B) \& \sim P(A \& \sim B) \rightarrow \sim P(\sim B \& A \vee \sim B \& \sim A)$$

⁶⁰ See e.g., G. Guitián, “Is Finance a ‘Structure of Sin’?”, in *Scripta Theologica*, 2013, 45(2), pp. 301–34.

⁶¹ L. Greenhouse, “Supreme Court Refuses to Hear Torture Appeal.”, *The New York Times*, in <https://www.nytimes.com/2007/10/09/washington/09cnd-scotus.html> [accessed 20/12/2022].

⁶² G.H. von Wright, “Deontic Logic”, *cit.*, p. 12.

⁶³ The equivalence between that and “ $O(p \rightarrow q) \rightarrow (O(p) \rightarrow O(q))$ ” is guaranteed by the Law of Exportation/Importation.

To this, von Wright applies his Principle of Deontic Distribution—the second critical move—which reads thus:

If an act is the disjunction of two other acts, then the proposition that the disjunction is permitted is the disjunction of the proposition that the first act is permitted and the proposition that the second act is permitted⁶⁴.

In symbols: $P(\alpha \vee \beta) \leftrightarrow (P(\alpha) \vee P(\beta))$.

As a result he obtains this:

$$\sim(P(\sim A \ \& \ B) \vee P(\sim A \ \& \ \sim B)) \ \& \ \sim P(A \ \& \ \sim B) \rightarrow \sim(P(\sim B \ \& \ A) \vee P(\sim B \ \& \ \sim A))$$

which, after applying De Morgan Law to the first conjunct of the protasis and to the apodosis, reads as

$$\sim P(\sim A \ \& \ B) \ \& \ \sim P(\sim A \ \& \ \sim B) \ \& \ \sim P(A \ \& \ \sim B) \rightarrow \sim P(\sim B \ \& \ A) \ \& \ \sim P(\sim B \ \& \ \sim A)$$

which, with these substitutions:

$$\sim P(\sim A \ \& \ B) = p$$

$$\sim P(\sim A \ \& \ \sim B) \equiv \sim P(\sim B \ \& \ \sim A) = q$$

$$\sim P(A \ \& \ \sim B) \equiv \sim P(\sim B \ \& \ A) = r$$

turns out to be a special case of $p \ \& \ q \ \& \ r \rightarrow r \ \& \ q$, which is a tautology of sentential calculus.

However, “ $\sim A \ \& \ B \vee \sim A \ \& \ \sim B$ ” (the perfect disjunctive normal form of “ $\sim A$ ”) really synonymous with “ $\sim A$ ”? That is, is $\sim A$ really the same action as $\sim A \ \& \ B \vee \sim A \ \& \ \sim B$? Is $\sim P(\sim A)$ equivalent to $\sim P(\sim A \ \& \ B \vee \sim A \ \& \ \sim B)$? Or: is $P(C)$ equivalent to $P(C \ \& \ D \vee C \ \& \ \sim D)$? What if C is an act apt to be permitted and has been permitted, but the other act, D , in the same “universe of discourse” as C generates a “predicament” like Jephthah’s⁶⁵ or Sophocles’ Antigone: both D and $\sim D$ is wrong, so that neither $C \ \& \ \sim D$ nor $C \ \& \ D$ look very apt to be permitted. But C alone does! Now, in his somewhat condescending answer to McLaughlin—even the greatest philosophers and dignified logicians are just humans—von Wright seems to hold that there is no such thing as C being permissible or not *all alone and by itself*, in isolation from other acts, unless, of course, they are the only acts in the given deontic space⁶⁶. The normal disjunctive forms are exactly for taking into account this fact.

⁶⁴ G.H. von Wright, “Deontic Logic”, *cit.*, p. 7.

⁶⁵ *Judges* 11, 30-39.

⁶⁶ G.H. von Wright, “A Note on Deontic logic...”, *cit.*, p., 508.

Yet, one who would like to employ deontic logic *à la* von Wright will typically have much more acts to mention within the scope of her deontic operators and so the tool replacing their names with the respective normal disjunctive forms will not turn out well adapted to the task. One will wish to know whether one may, e.g., cut down this tree, full stop, *not* whether one may do that *and* hunt down that wolf *and* send one's grandmother to the nursery home *or* do that *and not* to cut down that tree *and* hunt down the wolf all the same *or* send one's granny to the nursing house nevertheless... and so on. In "real life" the "universe" of all possible acts that need to be taken into account for deciding whether one of them is to be permitted or made obligatory is fairly large, and in addition, they are not logically independent of one another (posting the letter or burning *it*, as distinct from burning some other letter, declaring war on a country *by* posting a letter and the like), and the more we know about their logical intertwinings the more, as a rule, new facts must be admitted to our "universe of discourse"⁶⁷ ...the list of complaints about von Wright's methodology and the law(s) he thinks he can establish by its means is hardly closed.

The Principle of Deontic Distribution is not free from dubious facets, either. If I know that I may α or β , but don't know which, yet prefer β to α , I will go for β (even though I originally was licensed to α) and get away with it.

For the moment, I do not know how to deal with these difficulties. Perhaps the best choice is to give up SDL and adopt a dyadic logic sketched in von Wright (Von Wright 1956). As regards the paradoxes, Castañeda proposed a solution to all of them in 1981⁶⁸; however, his solution doesn't seem to have percolated all the way down to the nethermost regions of deonticians coping with the dirt of "real life".

Yet, as we know, *labor omnia vincit*⁶⁹, so let us persevere!

⁶⁷ A declaration of war by country X on country Y takes more than the existence of both countries and a speech-act, preferably in the written form (handing in a letter) to be legally valid. It takes also the recognition by country Y. The strange case of the Polish declaration of war on Japan in 1941 which Japan refused to recognize shows that this recognition need not be regarded as obligatory. See S. Dowell, "Strangest war in history? Seventy-one years ago today Poland declared WAR on its old friend Japan - and it lasted for 16 years", in <https://www.thefirstnews.com/article/strangest-war-in-history-seventy-one-years-ago-today-poland-declared-war-on-its-old-friend-japan---and-it-lived-for-16-years-3723> [accessed 20/12/2022]; M. Hiltz, "Poland Once Declared War On Japan – It Lasted 16 Years", in <https://www.warhistoryonline.com/war-articles/remember-when-poland-declared-war-on-japan-and-it-lived-16-years.html> [accessed 20/12/2022].

⁶⁸ H.N. Castañeda, "The Paradoxes of Deontic Logic: The Simplest Solution to all of them in one Fell Swoop", in R. Hilpinen, (ed.), *New Studies in Deontic Logic*, Synthese Library, 152, Springer, Dordrecht, 1981.

⁶⁹ Vergil, *Georgica*, I, 145. Mostly "*vincit*" in quotations, sometimes "*vincet*".